Internship (Japan) Report

Graduate School of Science, Department of Physics
MERIT 8th
Takuya Shimazaki

1. Outline of Internship

   Place: Mizuho-DL Financial Technology

   Period: Sep. 24, 2021 – Oct. 26, 2021

   Theme: Accuracy verification of an analytical approximation formula for discrete observation lookback options

2. Outline of Workshop

   Mizuho-DL Financial Technology provides Ph. D. students and post doctors with an internship. They can learn how their knowledge or skills are of great use in business. My theme is “Accuracy verification of an analytical approximation formula for discrete observation lookback options” using Monte-Carlo method.

3. Contents of Workshop

   Derivatives are financial products whose value is determined by original assets, e.g., stocks, interest rates, or currencies. For example, options are one kind of derivatives. Options are contracts to buy/sell the right to purchase financial assets at a specified price on a specific date. I consider lookback options thorough this workshop. The payoff function of a lookback option is dependent on not only price of an original asset at certain time but also its historical fluctuation. There are two kinds of lookback options. One is a continuous observation lookback option and the other is a discrete observation lookback option. It is known that there exists an analytical formula to price continuous observation lookback options when the Black-Scholes model is employed. On the other hand, discrete observation lookback options have no analytical formula. Therefore, we need to make use of analytical approximation formula or numerical simulation to evaluate the present value of discrete observation lookback options. Based on Monte Carlo method, the present value can be evaluated at arbitrary high accuracy,

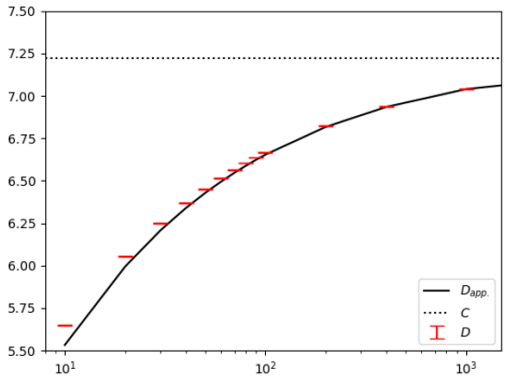

Fig. 1 Comparison of two methods

but it requires long time. This should be avoided because a quick response to a customer is often more vital. My aim in this workshop is to evaluate how accurate an analytical approximation formula works compared with Monte Carlo method.

Figure.1 shows the comparison between results of two methods, Monte Carlo method (red plots) and analytical approximation formula (black solid line). The vertical (horizontal) axis represents the present value of lookback option (the number of observational dates, respectively). When the division number increases, the two results agree with each other. Both asymptotically reach at a present value of continuous observation lookback option shown by the black dotted line.

So far, we implicitly assume that we evaluate the present value at observational date. The approximation formula is improved by extending the division number from natural number to rational number. Figure.2 shows the comparison between results of improved approximation formula (red plots) and Monte Carlo method (black solid line). The vertical (horizontal) axis represents the present value (date, respectively). It is noted that observational dates are on 1st of every month. The improved formula reproduces numerical results within the same error of the original approximation formula.

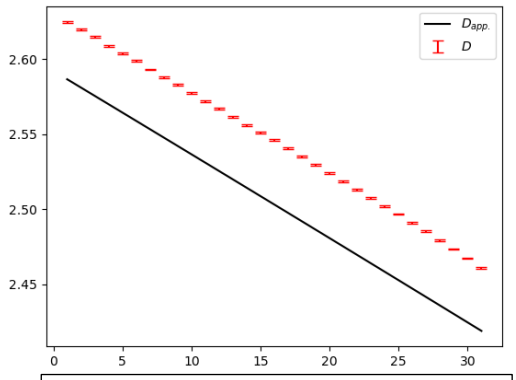

Fig. 2 Improved analytical formula

4. Acknowledgements

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