

# Report for MERIT Long-Term Overseas Dispatch

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**Place:** Jagiellonian University

**Group to visit:** Kamil Korzekwa's Group

**Period:** From 1 Jul. 2023 to 30 Jul. 2023

**Research project:** Study on the state conversion condition under symmetric operations

## Overview

I considered the problem of state convertibility via symmetric channels commuting with time translation with Prof. Kamil Korzekwa in Jagiellonian University in Poland. First, we showed that this problem can be formulated as a semidefinite programming, and thus it can be solved numerically. Moreover, we also clarified the analytical condition for the state convertibility for some specific cases, and confirmed that the condition coincides to the numerical one.

## Research

Energy conservation plays an important role not only in classical thermodynamics but also in quantum thermodynamics. In fact, the coherence nonincreasing property is derived from energy conservation, and more generally, the property follows from the fact that the operations allowed in quantum thermodynamics commutes with time translation. Then, We considered the state convertibility problem via this class of time-translation covariant completely positive and trace preserving (CPTP) maps.

We divided the problem into the following two cases, since the difficulty of the convertibility problem via the time-translation covariant CPTP maps greatly depends on the distribution of the energy levels. First, we considered the case where the energy gaps are different from each other. In this case, the transfer of coherence is simple, and we were able to get the condition for the conversion between multi-level systems. Second, we considered the opposite case, where the systems have equal energy gaps. In this case, the additional degrees of freedom in the transfer of coherence make the problem more complicated, but we revealed the convertibility condition from a two-level system to a



Fig. 1: Physics building of Jagiellonian University.

multi-level system.

As the research method, we made use of the Choi-Jamiołkowski isomorphism as an important idea. This enables us to transform the condition for the existence of channels into that of vectors. Moreover, we had Mr. Moisés Bermejo Morán to write the program to solve the transformed condition, which is written as a semidefinite programming. Thanks for him, we were able to numerically confirm the analytical result, and efficiently make progress.

I am going to write a paper about this result, and I would like to clarify the convertibility condition in the inverse direction, i.e., the conversion from a multi-level system to a two-level system. I would also get some insights into the conversion between multi-level systems.

### **Life**

The temperature in Kraków was about  $25^{\circ}\text{C}$  to  $30^{\circ}\text{C}$ , and the sunlight was a little stronger than in Japan.

While the literature campus of Jagiellonian University is in the center of Kraków, the buildings of science departments are located in the new campus, which is 40-minute tram ride far from the city center (see Fig. 1).

As for the schedule in a day, I did my work individually in the morning, went for lunch at 1 p.m. with the group members, had a short coffee break, and discussed with Prof. Korzekwa after that basically every day. When he was out or busy, I interacted with the group members about research, and confirmed whether the analytical result coincides with the numerical one with Mr. Morán.

On a holiday, I had Mr. Morán to guide around the city center of Kraków, and we saw the historic cityscape and Wisła River, which flows in Kraków (see Fig. 2). Kraków is the place Banach comes from, who is well known in analysis, and we saw the statues of Banach and Steinhaus in the park near the literature campus of Jagiellonian University (see Fig. 3).



Fig. 2: Wisła River flowing in Kraków.



Fig. 3: Statues of Banach and Steinhaus.

### **Acknowledgments**

I appreciate Prof. Korzekwa for accepting me as a visitor and always kindly discussing with me. I appreciate the group members for their warm welcome and the interaction with me. I especially appreciate Mr. Morán for writing the program for numerically confirming the state convertibility

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